HEAT AND MASS TRANSFER IN A SUBMERGED AXISYMMETRIC NON-SELF-SIMILAR JET

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The formulation and first solution of the problem of the development of a laminar submerged axisymmetric viscous incompressible fluid jet is due to Loitsyanskii [1]. The problem was solved on the basis of the laminar boundary-layer equations by the method of asymptotic expansions. Loitsyanskii found the first and second terms of the velocity component expansions in finite form. The third and fourth terms of the velocity of the expansions were determined in [2, 3], respectively. The solution permits taking account of the influence of the shape of the initial escape velocity profile on the velocity distribution in the jet. Heat transfer in an axisymmetric non-self-similar jet is examined in [4] for Pr = 1.

In this paper, the excess temperature distribution in submerged axisymmetric jets is obtained for any values of the Prandtl number, and certain features of the asymptotic velocity and temperature expansions are clarified within the framework of the Loitsyanskii theory. Results are presented of experimental investigations of the distribution of the gas impurity concentration in an axisymmetric turbulent air jet with initial nonuniformity in the escape velocity, which are compared with the solution obtained.

1. Laminar Jet

The heat-transport equation in a laminar viscous incompressible fluid boundary layer in the case of axisymmetric motion in a cylindrical coordinate system has the form

$$u\frac{\partial \Delta T}{\partial X}+v\frac{\partial \Delta T}{\partial r}=a\left(\frac{\partial^2 \Delta T}{\partial r^2}+\frac{1}{r}\frac{\partial \Delta T}{\partial r}\right). \quad (1)$$

Here X is the longitudinal distance from the jet source and r is the distance from the jet axis.

In addition to the condition of conservation of the momentum K_0 [1], an integral invariant of the problem is the condition of conservation of the excess heat content in the jet [5]

$$Q_0=2\pi\rho c_p\int_0^\infty u\Delta T r dr=\text{const}. \quad (2)$$

Let us introduce the new independent variables [1]

$$x=X,\quad \eta=r(X\nu)^{-1} \quad (3)$$

and let us add the expansion

$$\Delta T=\frac{d_1}{x}+\frac{d_2}{x^2}+\frac{d_3}{x^3}+\ldots \quad (4)$$

to the asymptotic expansions of the velocity and pressure components of the "dynamic" problem

$$u=\frac{\bar{a}'}{\eta}\frac{1}{x}+\frac{a_0'}{\eta}\frac{1}{x^2}+\frac{a_1'}{\eta}\frac{1}{x^3}+\frac{a_2'}{\eta}\frac{1}{x^4}+\ldots,\qquad v=\frac{\sqrt{\nu}}{x}\left\{\bar{a}'-\frac{\bar{a}}{\eta}+a_0'\frac{1}{x}+\left(a_1'+\frac{a_1}{\eta}\right)\frac{1}{x^2}+\ldots\right\}$$

Here $\bar{a}$, a_0, a_1, a_2 are functions defined in the papers mentioned, d_i (i = 1, 2,...) are unknown functions of η. As a result of substituting the expansions into (1) and equating co-

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efficients of terms containing identical powers of x, we obtain a system of ordinary differential equations to determine the unknown functions d_1, d_2, d_3,...:

$$d_1'' + \frac{1+\Pr\bar{a}}{\eta} d_1' + \Pr\frac{\bar{a}'}{\eta} d_1 = 0 ,$$

$$d_2'' + \frac{1+\Pr\bar{a}}{\eta} d_2' + 2\Pr\frac{\bar{a}'}{\eta} d_2 = -\Pr\frac{a_0'}{\eta} d_1 , \tag{5}$$

$$d_3'' + \frac{1+\Pr\bar{a}}{\eta} d_3' + 3\Pr\frac{\bar{a}'}{\eta} d_3 = -2\Pr\frac{a_0'}{\eta} d_2 - \Pr\frac{a_1'}{\eta} d_1 + \Pr\frac{a_1}{\eta} d_1' , \ldots$$

Analogously, from (2) we have the integral conditions

$$\int_0^\infty \bar{a} d_1 d\eta = \frac{Q_0}{2\pi\mu cP} , \quad \int_0^\infty (\bar{a}' d_2 + a_0' d_1)\, d\eta = 0 ,$$

$$\int_0^\infty (\bar{a}'\, \bar{d}_3 + a_0' d_2 + a_1' d_1)\, d\eta = 0 , \ldots \tag{6}$$

According to the boundary conditions of the problem [5]:

$$d_i(0) < M \text{ (bounded)}, \; d_i(\infty) = 0 \; (i = 1, 2, \ldots) . \tag{7}$$

In the new variable ζ [1]

$$\zeta = \frac{1}{4}\alpha^2\eta^2\left(1 + \frac{1}{4}\alpha^2\eta^2\right)^{-1}, \quad \alpha = \sqrt{\frac{3K_0}{16\pi\mu}} \tag{8}$$

the first equation in (5) is hypergeometric

$$\zeta(1-\zeta) d_1'' + [1 - 2(1-\Pr)\zeta] d_1' + 2\Pr d_1 = 0 ,$$

whose solution satisfying the boundary (7) and first integral conditions (6) has the form

$$d_1(\zeta) = \bar{\alpha}^2 F(-2\Pr, 1, 1, \zeta) = 2\bar{\alpha}^2 (1-\zeta)^{2\Pr} . \tag{9}$$

Here

$$\bar{\alpha} = \sqrt{\frac{Q_0(1+2\Pr)}{16\pi\mu c_P}} . \tag{10}$$

The second and third equations of (5) are

$$\zeta(1-\zeta) d_2'' + [1 - 2(1-\Pr)\zeta] d_2' + 4\Pr d_2 = \frac{\bar{\alpha}^2}{2}\Pr(1-\zeta)^{2\Pr}(1-4\zeta) ,$$

$$\zeta(1-\zeta) d_3'' + [1 - 2(1-\Pr)\zeta] d_3' + 6\Pr d_3 = -\frac{\beta^2\bar{\alpha}^2}{2}\Pr\left[\frac{3}{2} - (3\Pr+9)\zeta + (12\Pr+6)\zeta^2\right] . \tag{11}$$

The homogeneous conditions corresponding to (11) are hypergeometric

$$\zeta(1-\zeta) d_k'' + [1 - 2(1-\Pr)\zeta] d_k' + 2k\Pr d_k = 0 , \quad k = 1, 2$$

and have the solutions [6]

$$d_k = c_1 F(a_k, b_k, 1, \zeta) + c_2 \Bigg\{ F(a_k, b_k, 1, \zeta)\ln\zeta +$$

$$+ \sum_{n=1}^{\infty} \zeta^n \frac{(a_k)_n (b_k)_n}{(n!)2}[\psi(a_k+n) - \psi(a_k) + \psi(b_k+n) - 2\psi(n+1) + 2\psi(n)]\Bigg\} , \quad k = 2, 3 . \tag{12}$$

Here

$$a_k = \frac{1}{2} - \Pr + \frac{1}{2}[1 + 4(2k-1)\Pr + 4\Pr^2]^{1/2} ,$$

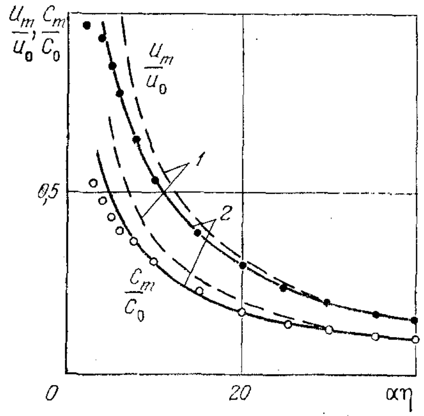

Fig. 1. Change in the maximal velocities u_m/u_0 and concentrations of the gas impurity C_m/C_0 along the $\bar{x}$ axis of an axisymmetric turbulent jet: 1) self-similar solution; 2) non-self-similar solution; points are results of measurements.

$$b_k = \frac{1}{2} - \mathrm{Pr} - \frac{1}{2}[1 + 4(2k-1)\mathrm{Pr} + 4\mathrm{Pr}^2]^{1/2};$$

ψ with the appropiate argument is the Euler ψ-function, and c_1 and c_2 are constants of integration. Particular integrals of the inhomogeneous differential equations (11) are

$$d_2(\zeta) = -\frac{\beta\bar{\alpha}^2}{2}(1-\mathrm{Pr})^{2\mathrm{Pr}}(1-4\mathrm{Pr}\zeta),$$

$$d_3(\zeta) = \frac{\beta\bar{\alpha}^2}{2}(1-\mathrm{Pr})^{2\mathrm{Pr}}\left[\mathrm{Pr}(2\mathrm{Pr}+1)\zeta^2 - \frac{5}{2}\mathrm{Pr}\zeta + \frac{1}{4}\right]. \tag{13}$$

According to the boundary conditions (7), the constants of integration are $c_1 = c_2 = 0$, and the solutions of (11) are only the particular integrals (13).

According to (8), expansion (4) in the variable η has the form

$$\Delta T = \frac{2\bar{\alpha}^2}{\left(1+\frac{1}{4}\alpha^2\eta^2\right)^{2\mathrm{Pr}}}\frac{1}{x}\left[1 - \frac{\beta}{4}\frac{1+\left(\frac{1}{4}-\mathrm{Pr}\right)\alpha^2\eta^2}{1+\frac{1}{4}\alpha^2\eta^2}\frac{1}{x}\right.$$
$$\left. + \frac{\beta^2}{16}\frac{1+\frac{1}{2}(1-5\mathrm{Pr})\alpha^2\eta^2 + \left(\frac{1}{2}\mathrm{Pr}^2 - \frac{3}{8}\mathrm{Pr} + \frac{1}{16}\right)\alpha^4\eta^4}{\left(1+\frac{1}{4}\alpha^2\eta^2\right)^2}\frac{1}{x^2}\right]. \tag{14}$$

For the value Pr = 1, a change in ΔT expresses similarity of the temperature and velocity fields in a submerged fluid jet, which corresponds to the results obtained earlier [4].

2. Turbulent Jet

Following the hypothesis of Loitsyanskii [1], an axisymmetric turbulent submerged viscous incompressible fluid jet can be considered laminar but with a molar viscosity. Therefore, the results obtained for a laminar jet are assumed true for a turbulent flow also, but under the assumption that the magnitudes of the velocities are averaged in time, and we take the coefficients of molar viscosity A and kinematic turbulent viscosity ε instead of the coefficients of molecular viscosity μ and kinematic viscosity ν. In conformity with this, we replace the Prandtl number of the laminar flow by the "turbulent" Prandtl number Pr_t. The

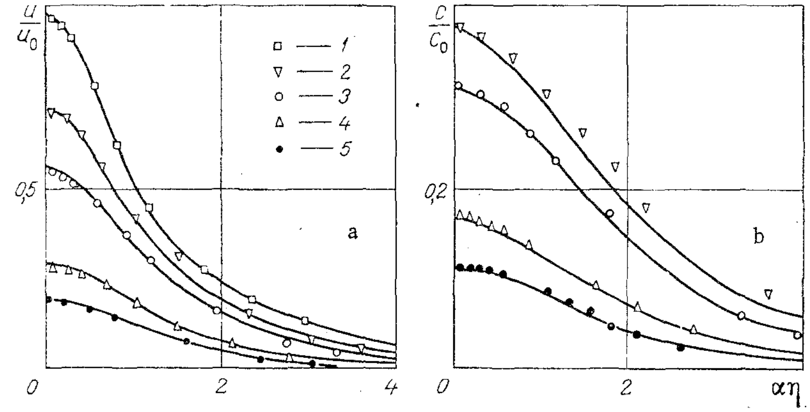

Fig. 2. Distribution of the velocity u/u_0 (a) and concentration C/C_0 (b) over sections $\alpha\eta$ of an axisymmetric turbulent jet: 1) $x/d = 6$; 2) 8; 3) 10; 4) 20; 5) 30.

intensity of mass transfer in axisymmetric turbulent submerged jets can be assessed by means of the distribution of the gas impurity concentration therein. Since the process of substance transfer in viscous incompressible fluid jets is equivalent to the process of heat transfer [5, 7], then these processes are described by identical equations with the appropriate replacement of the temperature ΔT by the expression for the concentration C, and the thermal diffusivity coefficient by the diffusion coefficient, i.e., Pr_t by Sc. Therefore, the distribution of the gas impurity concentration in turbulent axisymmetric submerged jets is described by (14) with the remarks made taken into account.

Experimental investigations were performed of the distribution of the gas impurity concentration in an axisymmetric turbulent air jet escaping from a nozzle ∅ 10.5 mm ($\mu_0 = 77$ m/sec, $Re = 4.9 \cdot 10^4$). A gas mixture, for which methane was used in a quantity of 1% relative to the air flow through the nozzle, was added to the pipeline supplying air to the nozzle. The air–gas jet obtained was delivered to a cylindrical pipe 800 mm in diameter, which connected to an exhaust fan, where the relationships between the air flow through the nozzle and the pipe were 1:10. The ratio between the diameters of the nozzle and the cylindrical pipe was such that the section of the jet from the nozzle exit to the section at 40 calibers distance could be considered a submerged jet. To eliminate the influence of the walls on development of the jet through the pipe, a fan continuously exhausted the air out at a rate of 0.5 m/sec. A Pitot-Prandtl tube was used as primary measuring device, and after having measured the velocity, was used as gas eliminator. The secondary measuring devices were MMN-250 micromanometers and the gas chromatograph "Color" with the recorder KSP-4. The accuracy of measuring the methane concentration in the mixture was 3%.

The integral constants of the jet investigated α and $\bar{\alpha}$ were determined according to (8) and (10) ($\alpha = 49.7$; $\bar{\alpha} = 15.8$, where Sc = 0.7 [7] was used in the latter formula in place of Pr). The characteristic constant β, which takes account of the influence of the initial velocity profile (at the nozzle exit), is determined by a semiempirical method [3] ($\beta = 63$).

The change in the maximal velocities u_m/u_0 and the concentrations C_m/C_0 along the jet axis $\bar{x} = x/d$ is represented in Fig. 1. Curve 1 corresponds to the self-similar solution (the first term for $\alpha\eta = 0$ is taken in the velocity and concentration expansions (14)), 2 is taken from the appropriate formulas of the non-self-similar solution (taking three terms of the expansions into account). It is seen from the figure that the section in which curves 1 and 2 practically merge can be considered the boundary of the transition and basic (self-similar) sections of the jet. For the jet investigated, this section is at a distance from the nozzle exit corresponding to $\bar{x} = 20$.

Results of calculating the distributions of the velocity u/u_0 (from formulas in [1, 2]) and the concentration C/C_0 (formula (14)) over the sections $\alpha\eta$ of the transition ($\bar{x} = 8.1$) and the basic ($\bar{x} = 20.3$) sections of the jet investigated, where the variable is related to the jet radius by the formula [3]: $\eta = K_\eta r_0/\bar{x}$ (for the jet investigated $k_\eta = 1.5$), are presented in Fig. 2a, b. The points represent results of measurements.

NOTATION

x, r, longitudinal and transverse coordinates; u, v, axial and radial components of the velocity vector; ΔT, excess temperature above the environmental; a, thermal diffusivity coefficient; K_0, momentum; Q_0, heat flux; c_p, specific heat at constant pressure; C, value of the impurity concentration; ρ, density; F, symbol for the hypergeometric function; Pr, Prandtl number; Re, Reynolds number; Sc, Schmidt number; u_0, value of the mean discharge velocity; C_0, value of the initial methane impurity concentration.

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VISCOUS FLOW OF LIQUID GLYCERIN—POLYETHYLENE GLYCOL—WATER MIXTURES

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The viscous flow of liquid water—glycerin mixtures is analyzed within the framework of free volume theory and thermodynamic reaction-rate theory.

Nonflammable liquids based on water—glycerin mixtures are currently widely used in industry and transportation [1]. These mixtures have the designations PGV, P-20, P-20M1, and P-20M2. The percentage content of the main components is shown in Table 1.

Measurements of the dynamic viscosity of these mixtures have been made in experiments for a broad range of temperatures and pressures [2]. Study of such mixtures is interesting first of all from the point of view of learning more about the intermolecular hydrogen bonds [3]. The molecules of all three components contain OH groups which form H-bonds between adjacent molecules. The high degree of molecular association resulting from this is the reason for the appreciable delay in transfer processes (diffusion, viscous flow, etc.) compared to nonassociated liquids of the same molecular weight.

Unfortunately, interpretation of experimental data in this case is complicated by the fact that the three-component mixture has a very complicated and nonuniform structure. Also, there are various additives ranging from 3 to 7% in the investigated mixtures, which may also affect the validity of any conclusions made. Finally, the interval of concentrations of the components in these mixtures is too narrow to obtain sufficiently reliable conclusions and recommendations as to the composition of mixtures with certain properties.

To analytically represent the empirical data, we will use a semiempirical relation linking the viscosity and density of the liquid and based on free volume theory [4]. Analysis of the data shows that it can be described satisfactorily if the empirical dependence of the specific volume of the "incompressible" nuclei on temperature is determined.

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